SUPPLEMENTARY MATERIALS FOR

CONSENSUS AROUND A COMMON DEFINITION OF ATLANTIC OVERTURNING WILL PROVIDE PROGRESS

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1. AMOC in depth space

The AMOC in depth space, AMOC(z), at given latitude y as a function of depth can be written as:

$$\mathrm{AMOC}(y,z) = \int_{-H}^{z} \left(\int_{x_W}^{x_E} v(x,y,z)\, dx \right) dz$$

where $v(x,y,z)$ is the meridional velocity at longitude x, latitude y, and depth z, H is the ocean depth.

2. AMOC in density space

The AMOC in density space, AMOC(ρ), at a given latitude y as a function of potential density can be written as:

$$\mathrm{AMOC}(y,\rho) = \int_{\rho}^{\rho_{\max}} \left(\int_{x_W}^{x_E} v(x,y,\rho)\, dx \right) d\rho$$

where $v(x,y,\rho)$ is the meridional velocity at longitude x, latitude y, and potential density ρ.

3. Zonal mean depth of isopycnals

To calculate the zonal mean depth of each isopycnal across a section, we follow these steps:

1. **Identify the isopycnal**: Select a specific isopycnal (e.g., 27.0) for analysis.
2. **Determine depths along the section**: For the selected isopycnal, identify its depth at every point along the entire longitudinal section from the western boundary (x_W) to the eastern boundary (x_E) at a given latitude y.

3. **Calculate the zonal mean depth**: Integrate the depths over the longitudinal section and normalize by the section length $L = x_E - x_W$.

Mathematically, this is expressed as:

$$\overline{z}(y, \rho_0) = \frac{1}{x_E - x_W} \int_{x_W}^{x_E} z(x, y, \rho_0)\, dx$$

where ρ_0 is the specific potential density value of the isopycnal (e.g., 27.0), $z(x, y, \rho_0)$ is the depth at which the density ρ_0 occurs at longitude x and latitude y, x_W and x_E are the longitudes of the western and eastern boundaries of the section, respectively.

4. Remapping AMOC from density space to depth space

To remap the AMOC from density space to depth space using the zonal mean depth of each isopycnal, we follow these steps:

1. **Calculate zonal mean depth of isopycnals**: As described in Section 3 then determine the zonal mean depth $\overline{z}(y, \rho)$ for each isopycnal ρ.
2. **Remap AMOC**: Integrate the AMOC in density space over the density layers and map these layers to their respective mean depths.

Mathematically, this is expressed as:

$$\mathrm{AMOC}_{\mathrm{remap}}(y, z) = \mathrm{AMOC}(y, \rho)\Big|_{\rho=\rho(y,z)}$$

where $\rho(y, z)$ represents the potential density at a given latitude y and depth z, which can be found using the zonal mean depth of each isopycnal:

$$\mathrm{AMOC}_{\mathrm{remap}}(y, z) = \mathrm{AMOC}(y, \rho_0) \quad \text{where} \quad z = \overline{z}(y, \rho_0)$$

This means that the AMOC transport in density space $\mathrm{AMOC}(y, \rho)$ is evaluated at the density value corresponding to the mean depth $\overline{z}$.