SUPPLEMENTARY MATERIALS FOR

Implosion in the Challenger Deep: Echo Sounding with the Shock Wave

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This supplement contains additional information on:

• Slant Range Uncertainty

• Propagated Time of Flight Uncertainty

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Slant Range Uncertainty

Recall the assumption that the horizontal separation between source and receiver was negligible and therefore the paths for the surface reflection and the surface-bottom-surface reflations were

$$p_s = (Z - z_1) + z_2, \tag{1}$$

$$p_{sbs} = (Z - z_1) + 2Z + z_2. \tag{2}$$

These paths may be combined to give the simplified result

$$\frac{\Delta t_{sbs-s}}{2}\overline{c} = Z, \tag{3}$$

where $\overline{c}$ is the effective sound speed over the entire path, Δt_{sbs-s} is the difference in time of arrival between the two paths, and Z, z_1 and z_2 are as described in the main text and shown in Figure 3. If we now consider a non-zero horizontal separation between the source and receiver, r, these equations, assuming that the seafloor and sea surface are effectively flat, become

$$p_{sT} = \sqrt{[(Z - z_1) + z_2]^2 + r^2}, \tag{4}$$

$$p_{sbsT} = \sqrt{[(Z - z_1) + 2Z + z_2]^2 + r^2}, \tag{5}$$

and

$$\Delta t_{sbs-s}\overline{c} = \sqrt{[(Z - z_1) + 2Z + z_2]^2 + r^2} - \sqrt{[(Z - z_1) + z_2]^2 + r^2}. \tag{6}$$

Letting $\alpha = [(Z - z_1) + 2Z + z_2]$ and $\beta = [(Z - z_1) + z_2]$ and rewriting Eq. 16 as

$$\Delta t_{sbs-s}\overline{c} = \alpha\sqrt{1 + (\frac{r}{\alpha})^2} - \beta\sqrt{1 + (\frac{r}{\beta})^2} \tag{7}$$

allows both square roots to be written as a Taylor's series where both $(r/\alpha)^2$ and $(r/\beta)^2 << 1$. Keeping only the 2nd order terms in both expansions and simplifying, equation 17 becomes

$$\Delta t_{sbs-s}\overline{c} = \alpha - \beta + \epsilon, \tag{8}$$

where

$$\epsilon = \frac{r^2}{2\alpha\beta}(\beta - \alpha) \tag{9}$$

represents the upper bound of the slant range correction term. ϵ gives the difference in the path traveled by p_{sT} and the surface reflected paths p_{sbsT} due to the separation between the source and receiver not accounted for in equation 12. Note that ϵ is always negative, since $\alpha > \beta$ is always true. Equations 17 and 18 further simplify and can be solved for the depth

Z, giving

$$\frac{\Delta t_{sbs-s}\overline{c}}{2(1-\frac{r^2}{2\alpha\beta})} = Z. \tag{10}$$

A comparison of equations 12 and 19 shows that $(1-\frac{r^2}{2\alpha\beta})$ gives the inverse of the fractional increase of the depth estimate by accounting for the additional path length due to the slant range. Note that as the horizontal separation of the source and receiver closes, r tends to zero and equation 19 becomes equation 12. Using the nominal values of Z = 10984 m, $Z - z_1$ = 9093 m, z_2 = 8259 m, and r = 626 m, the depth increase can be computed directly to 3 m or 99.97% of the nominal value. In practice, the horizontal separation is known with an uncertainty around O(1) and O(2) m. If we take a conservatively large uncertainty $\delta r = \pm 100$ m, a corresponding uncertainty in depth, δz_r = 1 m is generated. To account for the effects of the slant range we must increase our simplified estimate of depth by 3 ± 1 m. The uncertainty in horizontal range has been translated into an uncertainty in the depth estimate and is included in the overall uncertainty estimate through addition in quadrature.

Propagated Time of Flight Uncertainty

The depth of the Challenger Deep was determined by the modeled travel time, t_K, and the measured difference in time between two arrivals, Δt_{sbs-b}, according to,

$$t_K - \frac{\Delta t_{sbs-b}}{2} = 0. \tag{11}$$

The uncertainty comes from two main sources; the uncertainty in measured arrival time, t_{sbs-b}, and the uncertainty in the modeled travel time, t_K. The general form for the propagated uncertainty of independent and random uncertainties is;

$$\delta q = \sqrt{\Big(\frac{\partial q}{\partial x}\delta x\Big)^2 + ... + \Big(\frac{\partial q}{\partial z}\delta z\Big)^2}, \tag{12}$$

where q is the left hand side of equation 11 and x and z are the measured variables with uncertainties, δ and partial derivatives, ∂. The measured variables used to determine equation 11, are the arrival times measured by the hydrophones,t, temperature, T, salinity, S, pressure P and latitude, lat. t_K was a function of S, T, P and lat and Δt_{sbs-b} was a function of t. Applying equation 12 to equation 11 results in,

$$\delta q = \sqrt{\begin{array}{c}\Big(\frac{\partial t_K}{\partial T}\delta T\Big)^2 + \Big(\frac{\partial t_K}{\partial S}\delta S\Big)^2 + \Big(\frac{\partial t_K}{\partial P}\delta P\Big)^2 + \\ \Big(\frac{\partial t_K}{\partial lat}\delta lat\Big)^2 + \Big(\frac{\partial \Delta t_{sbs-b}}{\partial t}\delta t\Big)^2\end{array}}. \tag{13}$$

Recall from equation 9 in the main text that,

$$t_k = \sum_{i=I}^{K}\Big[\frac{\Delta z_i}{c_i}\Big], \tag{14}$$

where Δz_i is the width of the depth bin and c_i is measured sound speed at z_i, i is the starting depth of integration and K is the end depth, which can be expanded to,

$$t_K = \left[\frac{z_i}{c_i} - \frac{z_{i-1}}{c_i}\right] + \left[\frac{z_{i+1}}{c_{i+1}} - \frac{z_i}{c_{i+1}}\right] + \left[\frac{z_{i+2}}{c_{i+2}} - \frac{z_{i+1}}{c_{i+2}}\right] + ... + \left[\frac{z_K}{c_K} - \frac{z_{K-1}}{c_K}\right]. \tag{15}$$

In our case, the sound speed profile was sampled rapidly with a fine depth bin, thus the step-wise gradient is small and $\frac{z_i}{c_i} \approx \frac{z_i}{c_{i+1}}$, so Eq. 15 reduces to,

$$t_K = \frac{z_K}{c_K} - \frac{z_{i-1}}{c_i}. \tag{16}$$

For the modeled travel time from the surface to the bottom of the trench z_{i-1} is zero and only the first term remains. It follows that,

$$\frac{\partial t_K}{\partial X} = \frac{c_K \frac{\partial z_K}{\partial X} - z_K \frac{\partial c_i}{\partial X}}{c_i^2}, \tag{17}$$

where $X = T, S$ or P. The sound speed, c_i is not a function of lat and therefore the second term in numerator is zero for $\frac{\partial t_K}{\partial lat}$.

The modeled travel time, t_K relies on the empirical equation of state for sea water to determine Δz_i and the Del Grosso equation to determine c_i (IOC et al., 2010; Del Grosso, 1974). Del Grosso's empirically derived polynomial equation relies on regression fits to a library of previously collected data sets and describes their uncertainty as 0.05 m/s. The Del Grosso equation is a function of T, S and P. The empirical equation of state used to determine depth can be described by,

$$z = \frac{-2C}{B + \sqrt{B^2 - 4AC}}, \tag{18}$$

where A and B are functions of latitude and C is a function of $T, S,$ and P. Therefore, the uncertainty in Z can be summarized by,

$$\frac{\partial z}{\partial X} = \frac{-2C(g\frac{\partial C}{\partial X} - \frac{\partial (g)}{\partial X})}{g^2}, \tag{19}$$

where $g = B + \sqrt{B^2 - 4AC}$ and $X = T, S$ or P, and

$$\frac{\partial z}{\partial lat} = \frac{\partial B}{\partial lat} + \frac{B\frac{\partial B}{\partial lat} - 2C\frac{\partial A}{\partial lat}}{g - B}. \tag{20}$$

Combining equations 13, 17, 20, and 21 the uncertainty in t is,

$$\delta q = \sqrt{\left[\sum_{T,S,P} \Big(\frac{c_K \frac{\partial z_K}{\partial X} - z_K \frac{\partial c_i}{\partial X}}{c_i^2} \delta X\Big)^2\right] + \Big(\frac{c_K \frac{\partial z_K}{\partial lat}}{c_i^2} \delta lat\Big)^2 + \Big(\delta t\Big)^2}. \tag{21}$$

The uncertainties, $\delta T, \delta S$ and δP, are equal to the instrument uncertainty plus the uncertainties of the two models, such that

$$\delta X = \delta X_i + \delta z + \delta c, \tag{22}$$

where δX_i is the instrument uncertainty and δz and δc are the empirical equation of state and Del Grosso uncertainties respectively. The uncertainty in latitude, δlat was equal to the difference between the source deployment and recovery plus the uncertainty in the empirical equation of state. It was assumed that the uncertainty in the the difference between the two arrivals was fully constrained by the standard deviation of the four arrival time estimates (one for each hydrophone). The propagated uncertainty in q including the measurement uncertainty and the modeled uncertainty, was 3.7 ms seconds which corresponds to an uncertainty of $\pm$ 6 m. The uncertainty remains $\pm$ 6 m when adding in quadrature the slant range uncertainty (1 m) to the uncertainty in q.

References

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