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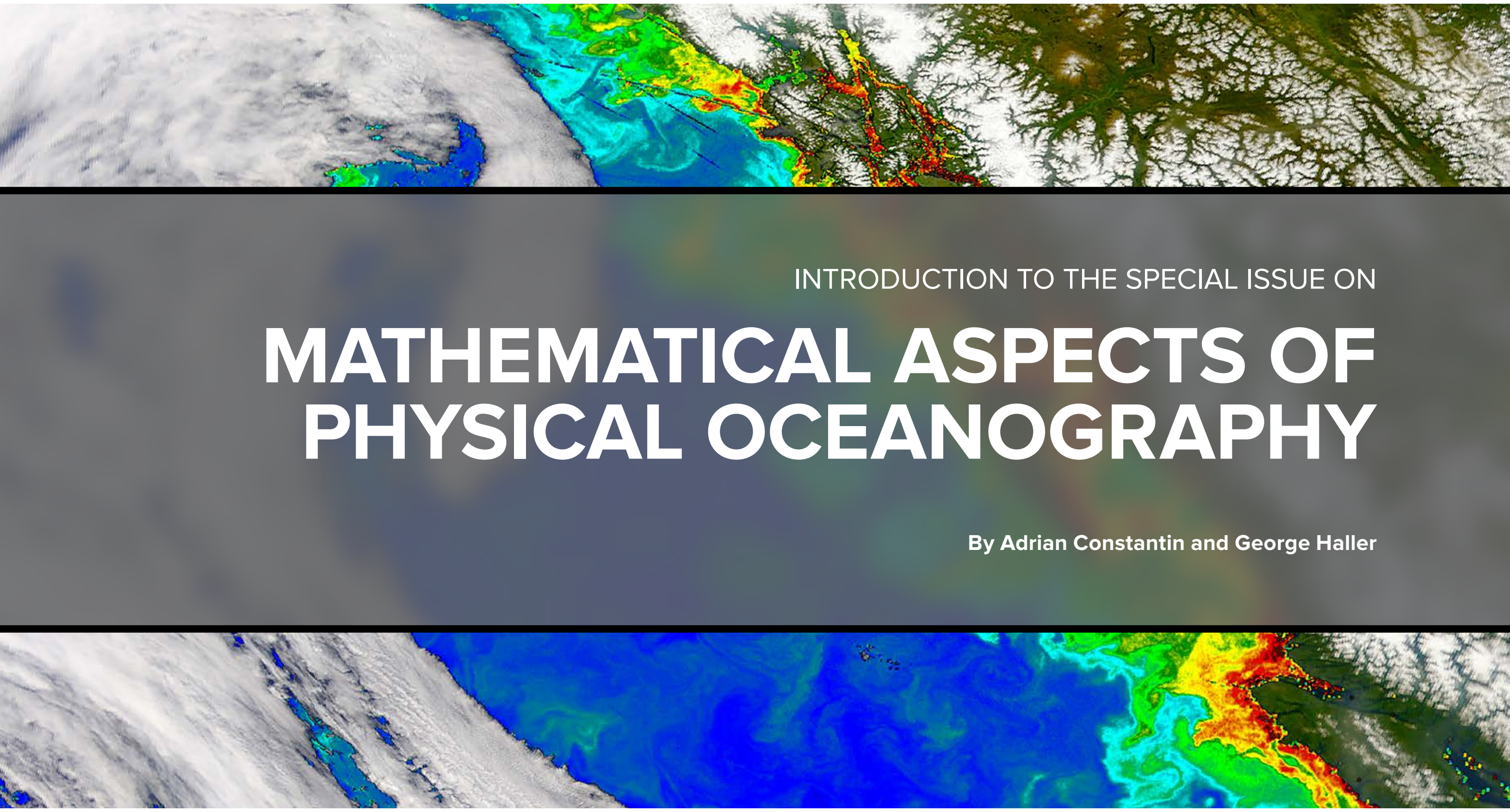

INTRODUCTION TO THE SPECIAL ISSUE ON

MATHEMATICAL ASPECTS OF PHYSICAL OCEANOGRAPHY

By Adrian Constantin and George Haller

Our knowledge and understanding of ocean dynamics is far from complete, but is expanding thanks in great part to new developments in mathematics. Some of the most important oceanographic discoveries have been made as a result of an integrated, multidisciplinary approach. The deepest understanding and the most interesting results almost always evolve from the interplay between theory and observation. A substantial body of theory to aid in the interpretation of observations has been developed, yet the ocean offers continually new data to challenge existing ideas—modern fieldwork is much more than cataloguing oceanic features, being designed as much to test theoretical hypotheses as it is to detect new phenomena.

The mathematical subject areas that are essential to the description of the changing spatiotemporal processes in the ocean are partial differential equations and dynamical systems. All subfields of physical oceanography rely heavily on these subjects, with analytical and computational aspects often intertwined and mutually reinforcing each other—their combined effect being stronger than the sum of each separate part. With a few notable exceptions, nonlinearity makes it impossible to obtain exact solutions to the governing equations for ocean flows. Consequently, numerical simulations play a prominent role in modern physical oceanography. However, the available technological means cannot cope with the vast range of temporal and spatial scales present in the ocean, and the prediction of fluid flow behavior becomes unrealistic if small disturbances draw energy from the main flow and subsequently grow rapidly until they become large enough to alter fundamentally the overall flow. An ongoing challenge is to reduce the computational problem to a manageable size, a task contingent upon making sensible simplifications that still provide accurate descriptions and predictions. This procedure often not only permits an in-depth study of a known phenomenon but sometimes also uncovers new processes that may not have been apparent or were overlooked. For successful derivation of adequate simplified models, it is necessary to understand the main ongoing mechanisms very well. This allows identification of physical regimes in which certain factors can be neglected, so that the dynamics is captured by a relatively simpler model. Such a model is amenable to in-depth theoretical studies that often reveal unexpected features and close the gap between real-world observations and idealized theoretical flow patterns.

The papers in this special issue reflect some new developments that were discussed during the program “Mathematical

Aspects of Physical Oceanography” that took place at the Erwin Schrödinger Institute for Mathematics and Physics (Vienna, Austria) from January 22 to March 23, 2018. Their authors discuss emerging theoretical methodologies and computational approaches, and describe high-precision experimental results.

By means of some general considerations and several case studies, **Johnson** comments on the pivotal role of systematic theoretical methods in physical oceanography. The papers by **Henry**, **Wheeler**, and **Basu** are devoted to equatorial ocean dynamics, which presents a rich variety of phenomena whose interplay is still to be fully understood, despite the availability of quite accurate field data (Johnson et al., 2001; Talley et al., 2011). The most significant of these phenomena are pronounced stratification and the presence of azimuthal currents with flow reversal (see the discussion in Constantin and Johnson, 2015), along with the prevalence of nonlinear dynamics (see the discussion in Boyd, 2018). **Henry’s** theoretical paper highlights the limitations of linear theory, while **Basu** presents numerical simulations for a recently derived nonlinear model for the equatorial currents in the Pacific (Constantin and Johnson 2017a), and **Wheeler** investigates equatorial wave-current interactions. While the β-plane approximation is accurate for equatorial flows, the discussions in Dellar (2011) and Paldor (2015) show that it is inconsistent for non-equatorial flows. This motivates the investigation of shallow-water large-scale ocean flows in a rotating frame in spherical coordinates. Using structural properties that become apparent in the shallow-water regime, solutions can be obtained that capture essential features of large-scale ocean gyres (Constantin and Johnson, 2017b), of general near-surface wind-induced ocean currents (as discussed in the paper by **Constantin and Johnson**), and of the Antarctic Circumpolar Current, as shown in the paper by **Haziot and Marynets**.

The vorticity of the fluid is an important flow characteristic (see Apel, 1987), being the hallmark of non-uniform currents (see Saffman, 1981), while coherent vortices are the building blocks of complex fluid flows (see the discussions in Haller, 2005, and Haller et al., 2016). The article by **Martin** discusses some vorticity aspects of large-scale ocean flows, presenting recent results as well as some open questions. Finally, by means of recent field experiments, **Christensen et al.** highlight some of the challenges in short-term predictions (hours to days) for oceanic drift trajectories. These authors emphasize the impact of surface waves because most buoyant objects are at or close to the ocean surface, so that the direct and indirect influences of surface waves are important.

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AUTHORS

Adrian Constantin (adrian.constantin@univie.ac.at) is Professor of Mathematics, University of Vienna, Austria. **George Haller** (georgehaller@ethz.ch) is Professor of Nonlinear Dynamics, Swiss Federal Institute of Technology Zürich, Switzerland.

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