Settling of Particles in Aquatic Environments

Low Reynolds Numbers

BY EMMANUEL BOSS, LEE KARP-BOSS, AND PETER A. JUMARS

PURPOSE OF ACTIVITY

The purpose of this activity is to familiarize students with how a particle's size, shape and orientation affects its settling at low Reynolds numbers. This activity can also be used to teach statistical skills (e.g., replication of measurements, propagation of error, type I vs. type II regressions).

AUDIENCE

Components of this activity have been used in a variety of classes, including an advanced graduate class on particle dynamics, a junior-senior undergraduate class on organism design, and a sophomore undergraduate class on physics for marine sciences at the School of Marine Sciences of the University of Maine (more information available at http://misclab.umeoce.maine.edu/education.htm). Students should be familiar with the concepts of Reynolds number and drag prior to the lab.

BACKGROUND

Settling of particles is the primary transport mode for carbon from the surface oceans to depth and is the physical process behind the "biological pump" that incorporates dissolved inorganic carbon into particulate structures of phytoplankton that later sink. Material that does not sink will eventually get remineralized or otherwise dissolved in the upper ocean. Settling is also an integral part of the life of planktonic organisms, regulating their vertical position relative to light, nutrients, prey, and predators. It plays an important role in sediment dynamics by, among other consequences, sorting the material arriving to the seabed and providing one mechanism for aggregation. The settling of small marine particles (phytoplankton, larvae, fine sediments) is a case of low-Reynolds-number flow. Humans have developed intuition for high (turbulent)-Reynolds-number flows, based on our own experience, but have very little intuition for the world of low Reynolds numbers. Yet, this is the world inhabited by the majority of planktonic organisms.

RESEARCH QUESTION

How does settling velocity depend on size, shape, and orientation at low Reynolds numbers? Does Stokes' solution hold, and over what range of Reynolds numbers?

HYPOTHESIS

Stokes' solution is applicable for settling at low Reynolds numbers ($Re << 1$). When particles are not spherical, deviations from that solution are expected; in general, the larger the cross-sectional area perpendicular to the settling direction, the slower the particle settles.

APPROACH

Students will measure settling velocities of a series of small beads of varying sizes, but which are all made of the same three materials (e.g., clay, steel, glass) in a highly viscous fluid before comparing results in water. The student will explore the effect of shape on settling by constructing models of non-spherical particles and measuring how their settling changes with orientation.

Emmanuel Boss *(emmanuel.boss@maine.edu) is Associate Professor, School of Marine Sciences, University of Maine, Orono, ME, USA.* **Lee Karp-Boss** *is Research Assistant Professor, School of Marine Sciences, University of Maine, Orono, ME, USA.* **Peter A. Jumars** *is Professor, School of Marine Sciences, University of Maine, Orono, ME, USA.*

MATERIALS

- Glycerin or clear corn syrup (Karo or other brand found in grocery stores).
- Glass beads and steel ball bearings of five different diameters spanning at least O(1 mm) to O(1 cm) (found in hardware stores). (O() means order of magnitude.) Be careful to obtain glass and steel of uniform specific gravity. The bigger the range of sizes the better.
- Thermally setting modeling clay (e.g., SCULPEY, found in craft stores).
- Large cylindrical or rectangular settling tanks. Based on observed and calculated flow patterns around spheres, the cylinder diameter would have to be roughly 100 times the diameter of the settling object to eliminate wall effects. Wall effects will slow the object's terminal sinking velocity, erroneously indicating a greater-than-actual fluid viscosity. At the precision sought in this laboratory exercise, a 2-liter graduated cylinder works very well for 1-mm beads and sufficiently well for beads as large as 1-cm.
- A balance.
- Stop watch(es).
- Calipers for measuring bead diameter.
- Meter stick to measure distance along the cylinder.

ACTIVITY

1. Measuring densities of fluids and solid bodies: Ask students to measure densities of the fluid and beads. (Have *them* figure out how.) (10 min)
2. Effect of size on settling (Figure 1): Ask students to measure settling velocities of a series of ball bearings or glass beads of different diameters (D). Emphasize to the students that the start-time for measurement is when the object has reached several centimeters below the surface (at a point where the beads have attained terminal velocity) and stop-time is several centimeters above bottom. (15 min)

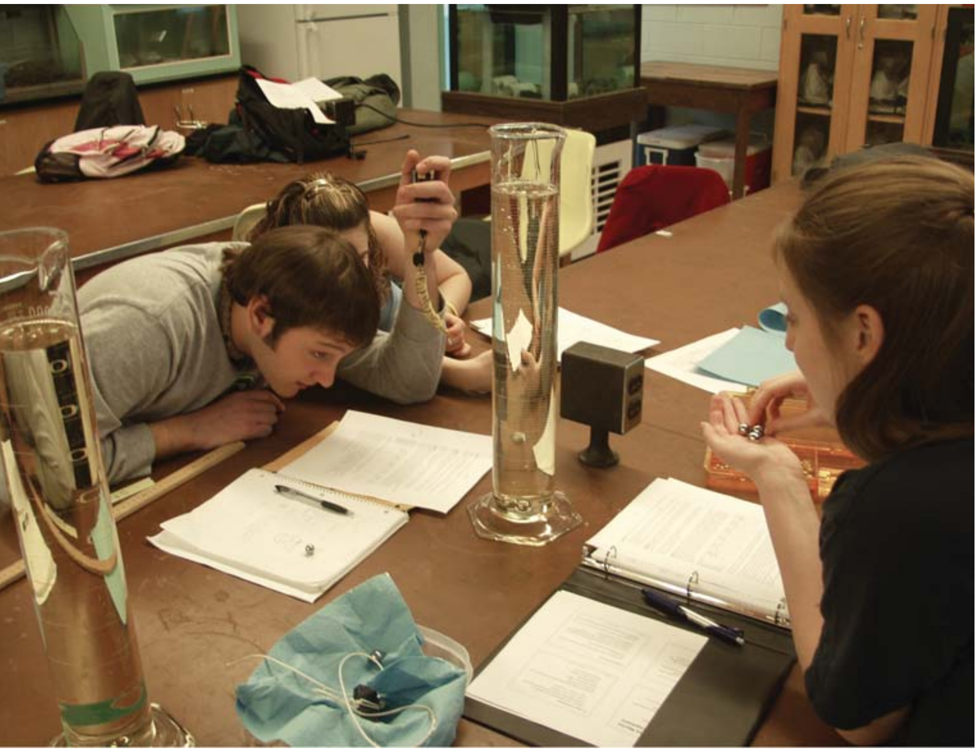

Figure 1. A student measuring sinking speed of a steel bead in a large, graduated cylinder. Photo credit: J. Loftin, University of Maine, Orono, during class SMS 204, spring 2006.

3. As *homework*, have students graph settling velocity as function of D and D^2. Which of the relationships is more linear? Which has an intercept closer to zero? The force balance for a settling particle (at terminal velocity) is: $F_{\mathrm{drag}} = M g - F_{\mathrm{buoyancy}}$ (where g is the gravitational acceleration, F_{drag} is drag force, M is the particle's mass and F_{buoyancy} is the buoyancy force ($V \rho_f g$, where V is the particle's volume), and thus $F_{\mathrm{drag}} = (\rho_p - \rho_f)\, g\, \pi\, D^3/6$. Given Stokes' equation (known as Stokes' law) for terminal velocity of a settling sphere ($w_p = (\rho_p - \rho_f)\, g\, D^2/18\mu_f$, where w_p is the sinking velocity, ρ_p and ρ_f are the densities of the particle and the fluid, respectively, and μ the fluid's dynamic viscosity), how is drag related to viscosity, settling velocity and particle size (Answer: $F_{\mathrm{drag}} = 3\, \pi\, \mu_f\, D\, w_p$)? Have students plot the drag force $(\rho_p - \rho_f)\, g\, \pi\, D^3/6$ as a function of $3\, \pi\, D\, w_p$. Is the value of the slope reasonable based on the fluid viscosity (Figure 2)? They can use published web content to find viscosities of glycerin and corn syrup, but beware that these viscosities are highly temperature dependent, and that the viscosity of corn syrup depends on the manufacturing process and absorbed water. Students are now in a position to

compute the Reynolds number $\{Re = \rho_f w_p D/ \mu_f\}$ of the flow past the particles. Stokes' equation for settling is valid only for $Re << 1$. Has this condition been satisfied in the experiment?

4. Using glass (or other not-very-dense) beads, perform a settling experiment in water and see whether Stokes' law provides a good approximation for settling at higher *Re* ($\mu_w \sim 10^{-3}$ N s m^{-2}) (15 min in class and analysis as *homework*).
5. Effects of shape on settling (Figure 3): Prior to the settling class, give each student nine pieces of modeling clay, each of equal mass. Ask them as a *homework* exercise to shape the clay into three perfectly round particles and six identical, non-spherical particles (or various groups of identical particles, each group being different from the others in a well described way, for example, a longer and thinner cylinder or a phytoplankton-like shape with more spines (*cf.* Padisák et al., 2003). Have the student bake the particles at home, following the manufacturer's instructions. In class (20 min): Ask the student to measure the clay's density and its settling velocity as a function of shape. Does orientation matter? If it does, what orientations produce the fastest and slowest sinking speeds? Do they maintain orientation in different fluids (with changing *Re*, try water)? Homework: How big should a sphere be to provide the same sinking velocity?

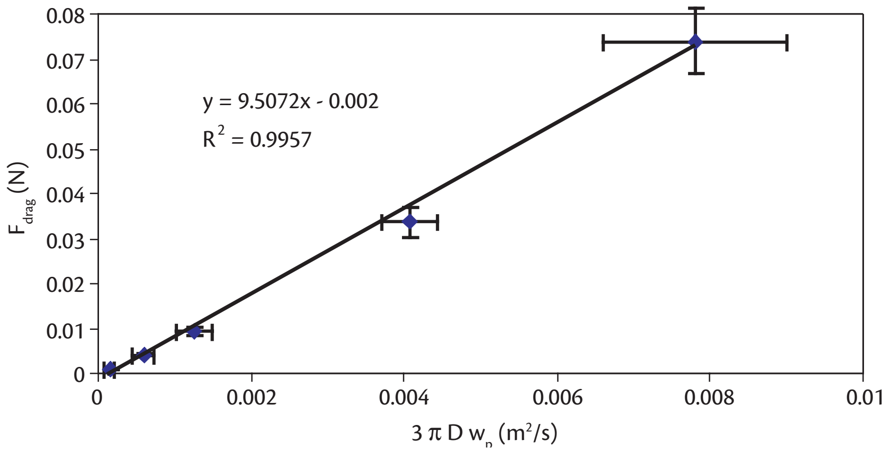

Figure 2. Drag force as function of $3\pi Dw_p$. Based on theory, we expect the intercept to be zero and the slope to be the fluid viscosity (here Karo syrup). The slope is consistent with published values. Data were collected by students on five different beads and are averages of those obtained by five independent groups of students.

SOME GENERAL COMMENTS ON THIS LAB

This lab is usually taught in a single hour. Doing the analysis (homework) exercises in class extends the class to about three hours. Significant wall effects are noticeable for 1-cm diameter particles (e.g., wp reduced by O(25%)). As a result, when students construct the plot to determine fluid viscosity, they may notice that the data points are fit more closely by a shallow parabola than by a line. If this observation is made, it can be used to generate a discussion of influence wall effects have on sinking rate. Molding non-spherical particles would add another hour to the lab if not done previously as homework. This lab works well for teams (3–4 students) dividing the tasks among themselves, checking each other's numbers, and discussing results and methods.

The fluid can be reused (siphoned out) as can the balls and clay particles (a wand with an attached magnet allows reuse of the steel beads within the same lab).

Setup is O(20 min), while clean-up time is O(1 h), in particular due to the long time it takes to siphon viscous fluids. (Use at least 1.2 cm diameter tubes.)

The lab can be embellished or modified in many ways depending on student backgrounds. For example, students can be asked to derive Stokes' law from the first two equations in the Activity section. The law applies equally well to positively buoyant, rigid, spherical particles, but fluid inclusions (bubbles or immiscible liquids) move faster because induced internal flows reduce friction with the surrounding fluid and hence the drag force. In a non-traditional approach with juniors and seniors, one of us uses low-*Re* settling of spheres to introduce the concept of drag forces, avoiding the confusion that often arises by starting with the more abstract idea of drag coefficients (more information available at http://www.marine.maine.edu/~jumars/classes/SMS_481/index.html). Graduate students can benefit from solving for relative flow velocity past the sphere as a function of location (in spherical coordinates centered on the sphere).

Figure 3. Measuring the sinking speed of clay models of plankton. Photo credit: J. Loftin, University of Maine, Orono, during class SMS 204, spring 2006.

RELATED TOPICS FOR FUTURE INVESTIGATIONS

- Drag and settling at high-*Re* flows
- Combining settling with dissolution
- Increased concentration of settling particles at density interfaces
- Hindered settling (modified settling in high concentrations of particles)

ADDITIONAL LITERATURE AND RESOURCES

Settling Activities Web Sites for High School and College Students

http://aslo.org/education/teaching/fluids.html

http://webusers.physics.umn.edu/~rlua/teaching/labdev/beadsyrup/

Multimedia Literature/ Resources

Taylor, G.I. 1967. *Low-Reynolds-Number Flows* (Encyclopaedia Britannica Educational Corp., Chicago). Video No. 21617. Parts of this film have been reproduced in: Homsey, G.M. 2000. *Multi-Media Fluid Dynamics.* Cambridge University Press, New York, NY. (Funded by National Science Foundation Division of Undergraduate Education.)

A relevant *Mathematica* notebook [Online] Available at: http://www.nd.edu/~mjm/creeping_sphere.nb

Some Fluid Dynamics Literature for Instructors or Grad Students

Batchelor, G.K. 1967. *An Introduction to Fluid Dynamics.* Cambridge University Press, Cambridge, United Kingdom.

Clift, R., J.R. Grace, and M.E. Webber. 1978. *Bubbles, Drops and Particles.* Academic Press, New York, NY. (Reprinted in 2005 by Dover.)

Leal, L.G. 1992. *Laminar Flow and Convective Transport Processes.* Butterworth-Heinemann, Boston, MA.

Oceanography-Related Literature

Dietrich, W.E. 1982. Settling velocities of natural particles. *Water Resources Research* 18: 1,615–1,626.

Ittekot, V., P. Schafer, S. Honjo, and P.J. Depetris, eds. 1996. *Particle Flux in the Ocean*. John Wiley & Sons Ltd. Sons Ltd., New York, N.Y., 367 pp. (Book chapters [Online] Available at: http://www.icsu-scope.org/downloadpubs/scope57/contents.html).

Komar, P.D., and C.E. Reimers. 1978. Grain shape effects on settling rates. *Journal of Geology* 86:193–209.

Padisák J., É. Soróczki-Pintér, and Z. Rezner. 2003. Sinking properties of some phytoplankton shapes and the relation of form resistance to morphological diversity of plankton—An experimental study. *Hydrobiologia* 500:243–257.

Richardson, T.L., and J.J. Cullen. 1995. Changes in buoyancy and chemical composition during growth of a coastal marine diatom: Ecological and biogeochemical consequences. *Marine Ecology Progress Series* 128:77–90.

Smayda, T.J. 1970. The suspension and sinking of phytoplankton in the sea. *Oceanography and Marine Biology Annual Review* 8:353–414.

Smetacek, V.S. 1985. Role of sinking in diatom life-history cycles: Ecological, evolutionary and geological significance. *Marine Biology* 84:239–251.

Villareal, T.A., C. Pilskaln, M. Brzezinkski, F. Lipschultz, M. Dennett, and G. Gardner. 1999. New estimates of vertical nitrate transport by migrating diatom mats. *Nature* 387:423–425.