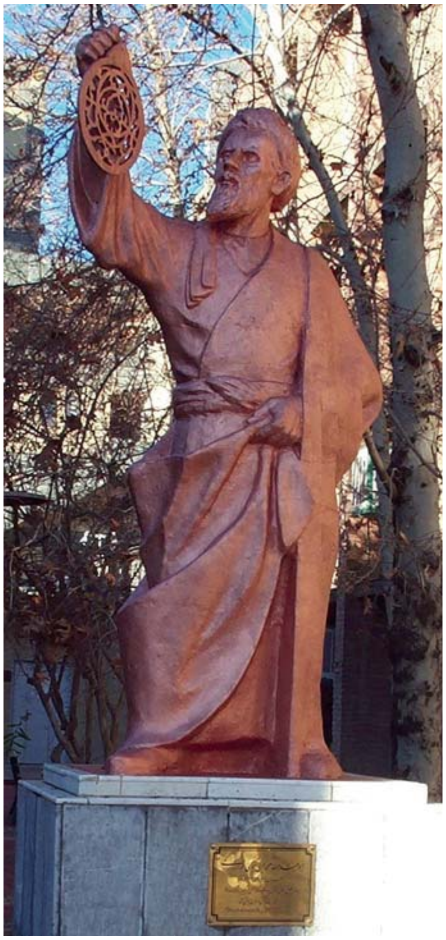
THE IMPORTANCE OF BEING *Quantitative*

BY MATTHIAS TOMCZAK

When I went to university forty years ago, I was taught the facts about the ocean and what lives in it, plain and simple. The method of instruction followed by my instructors was straightforward and can still be found in classical oceanography textbooks on library shelves: geomorphology of the seafloor; physical and chemical properties of seawater; water masses; ocean circulation; waves; tides; marine life; and so on. Discussion of the scientific method that produced the presented facts and reflection on the nature of scientific investigation was left to discussions with fellow students in the coffee shop, and in any case, it was not high on the agenda. Science was interesting as such and had not yet gained the tainted reputation of the years to come that resulted from nuclear accidents, ozone depletion, greenhouse warming, and other calamities.

Forty years on, every reputable textbook on marine sciences is prefaced by some words on the nature of scientific enquiry or tries to convey the essence of scientific discovery through a chapter on the history of oceanography. The change reflects the different attitude of students towards science that has been commented upon repeatedly in this column: Today's students are science skeptics—at least in the "developed world"—and have to be taught the very principles of objective scientific analysis from the fundamentals on up.



Statue of al-Khwarizmi in front of the Faculty of Mathematics of Amirkabir University of Technology in Tehran, Iran. Photo credit: M. Tomczak.

Today, any first-year science course has to be an induction course into the scientific method. If it is a marine science course, it will use the example of the ocean environment to achieve that goal. The course will communicate facts and results along the way, but never lose sight of the main task: to explain why science is not a matter of subjective belief, why science can be sure of its results within the boundaries where they underwent verification, and why it is perfectly acceptable to test these boundaries again and again.

In a nutshell, the aim of any first-year science course is to teach students how to think scientifically. This is not achieved by merely reading the preface of a book or going through the history of oceanography (although that may be a good start), it has to be part of the teaching agenda through the entire course.

An essential element of the scientific method is quantitative assessment. There are, of course, situations where a rough qualitative judgment is sufficient to disprove a theory—a quick look at the night sky invalidates the theory that the moon is blue. But, many situations re-

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quire quantification to determine whether a result predicted by science will be achieved or not. This is particularly true in the environmental sciences, where it is mostly impossible to study processes in isolation and where the signal of interest has to be extracted from a data set controlled by many factors.

Quantification is impossible without mathematics. There is no denying that the mathematical skills of today's students are low and that many students are convinced that anything that requires calculation is beyond their capabilities. This poses a real challenge to any science lecturer.

Hidden Knowledge Brought into Consciousness

As a lecturer of physical oceanography, I am particularly interested in breaking down real and imagined barriers between students and mathematics. The strength of the historical approach, particularly if it is expanded to the history of science in general, is that it can help to demonstrate to students how much they already know without noticing. I found much useful material for the task when I began to teach a class "Science, Civilization, and Society" (for course material, including descriptions of 35 lectures, see http://www.scienceandsociety.tk) a year ago. I discovered that the material I had to prepare for that course was well suited to convince my oceanography students that they possess abilities they did not think they had.

Begin with the simplest of tasks, the four basic operations of arithmetic. Demonstrate that your students are already masters in the application of one of the greatest inventions of the human mind, the place value number system: Ask them to multiply XIX by CCI. A tall order? Then ask them to multiply 19 by 201—even today's students, who grew up with desk calculators, should be able to find the answer in reasonable time.

Hopefully, the discovery of your students' ability to use a place value number system[1] with ease generates some curiosity about the origin of such an ingenious invention. Whether it is in a science lab or in the supermarket, Arabic numerals are so much easier to use than Roman numerals. How did the Romans use their absolute value number system for advanced calculations? By counting objects: "calculus" is Latin for "pebble," so to calculate means to count pebbles. How did Greek science progress without a place value number system? Remind your students of Pythagoras: He avoided arithmetic and concentrated on measuring lengths and areas, using a ruler, a divider, and geometry.

At this stage your students should be impressed to find that they are better trained in some elementary mathematical skills than Pythagoras and Archimedes (who contributed to oceanography by formulating the law of buoyancy, but also spent much time pondering without success about ways to write large numbers in the Greek number system). They owe their mathematical prowess to an Indian invention. The Sanskrit place value number system, first published in 458, allowed Indian scientists to become world experts in arithmetic. Harmonic functions were known in India before the 7th century. An opportunity to remind students of this achievement presents itself when you arrive at the discussion of the harmonic method of tide prediction: The interpolation formula to compute harmonic functions was published by Brahmagupta in 628.

The Sanskrit numerals have not changed much from the time of their first introduction to our use today. In the 9th century, Muslim scientists working in Baghdad raised the power of math-

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[1] In a place value number system the value of each numeral is determined by its place in the number. For example, the numeral 1 in 101 represents 100 at the front but 1 at the end. In contrast, numerals in absolute number systems always retain their value regardless of their position in a number. The C in CCI always represents 100.

ematics another notch by combining Greek geometry with Indian arithmetic. Their impact was so overwhelming that we now call the Indian Sanskrit numerals "Arabic" numerals. The name of one of the greatest Muslim mathematicians, Al-Khwarizmi (see photo), was immortalized in the word algorithm. His major work, *Kitab al-jabr wa l-muqabala* (literally, *The Book of Reduction and Comparison*), from which our word "algebra" derives, should be of great interest for the money-minded students of today; its opening chapter states:

The imam and emir of the believers, al-Ma'mun, encouraged me to write a concise work on the calculations al-jabr and al-muqabala, confined to a pleasant and interesting art of calculation, which people constantly have need of for their inheritances, their wills, their judgments and their transactions, and in all the things they have to do together, notably, the measurement of land, the digging of canals, geometry and other things of that kind.

Leibnitz, Newton, and the Need to be Quantitative

Physical oceanography is, of course, more than addition and multiplication, and differential equations can raise the anxiety level of the class. There comes a point when Newton's Laws cannot be avoided. The historical approach cannot teach students the essentials of differential calculus, but it can generate an appreciation of its value and purpose.

Start again with a simple problem. If a car travels at 50 km/h, how far has it traveled after two hours? How far after three? The law of constant motion was known to Aristotle. The Indian astronomer Bhaskara, who lived in the 12th century, was the first to formulate it in mathematical terms; in his discussion of the laws of motion he introduced quantitative assessment and described the velocity v of an object in steady motion as covered distance l divided by elapsed time t by using the equation $v = l / t$. But Newton's Laws are not about motion at constant speed, they are about forces that result in acceleration. To make use of the newly discovered laws of nature, science needed a tool to measure acceleration.

How do you measure acceleration? The petrolheads in your class will have the answer: You stop the time it takes to get from zero to 60 mph. But this is a rather crude measure and cannot legally be applied in a 30 mph zone. If we want to know the speed and position of a car at any arbitrary moment in time we need a method to determine speed from very small time increments. In other words, going back to Bhaskara, we have to write $v = dl / dt$ and make dt very small. Differential calculus was developed to meet that requirement. The fact that it was developed simultaneously by Gottfried Leibnitz in Germany and Sir Isaac Newton in England is evidence that the time had come where science could no longer do without it.

Why go to these lengths and talk about the history of differential calculus in a marine science class? Why not leave Newton's Laws to the physical oceanographers and enjoy the beauty of the marine environment in its geological and biological forms? What is the point in trying to turn all students of marine science into proficient mathematicians? It would be a misunderstanding if that were the intention. But we have to realize the fact that phobia of quantitative assessment is widespread in today's science classes, and quantification means calculation. Generating an appreciation of the value of mathematics is the first step in overcoming the phobia of being quantitative.

Sound, scientifically based environmental management is impossible without quantitative assessment. Whether it is the predictability of the tide or the complex evolution of an ecosystem, qualitative description can only be the start for serious scientific study. The next step is quantitative assessment, which cannot be achieved without mathematics. As future environmental consultants and managers, our students may well leave that step to a trained mathematician. One of the lessons they should take away from a marine science course is an appreciation of the place of mathematics in the environmental management process.

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